Package: PAGFL (via r-universe)

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Title Joint Estimation of Latent Groups and Group-Specific Coefficients in Panel Data Models

Version 1.1.1

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Description Latent group structures are a common challenge in panel data analysis. Disregarding group-level heterogeneity can introduce bias. Conversely, estimating individual coefficients for each cross-sectional unit is inefficient and may lead to high uncertainty. This package addresses the issue of unobservable group structures by implementing the pairwise adaptive group fused Lasso (PAGFL) by Mehrabani (2023) <doi:10.1016/j.jeconom.2022.12.002>. PAGFL identifies latent group structures and group-specific coefficients in a single step. On top of that, we extend the PAGFL to time-varying coefficient functions.

License AGPL (>= 3)

Encoding UTF-8

Roxygen list(markdown = TRUE)

RoxygenNote 7.3.2

LinkingTo Rcpp, RcppArmadillo, RcppParallel

Imports Rcpp, lifecycle, ggplot2, RcppParallel

BugReports https://github.com/Paul-Haimerl/PAGFL/issues

URL https://github.com/Paul-Haimerl/PAGFL

Suggests testthat (>= 3.0.0)

Config/testthat/edition 3

Repository https://paul-haimerl.r-universe.dev

RemoteUrl https://github.com/paul-haimerl/pagfl

RemoteRef HEAD

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Contents

`grouped_plm` *Grouped Panel Data Model*

Description

Estimate a grouped panel data model given an observed group structure. Slope parameters are homogeneous within groups but heterogeneous across groups. This function supports both static and dynamic panel data models, with or without endogenous regressors.

Usage

```
grouped_plm(
  formula,
  data,
  groups,
  index = NULL,
  n_periods = NULL,
  method = "PLS",
  Z = NULL,
  bias_correc = FALSE,
  rho = 0.07 * log(N * n_periods)/sqrt(N * n_periods),
  verbose = TRUE,
  parallel = TRUE,
  ...
)

## S3 method for class 'gplm'
print(x, ...)

## S3 method for class 'gplm'
formula(x, ...)

## S3 method for class 'gplm'
df.residual(object, ...)

## S3 method for class 'gplm'
summary(object, ...)
```

```
## S3 method for class 'gplm'
coef(object, ...)

## S3 method for class 'gplm'
residuals(object, ...)

## S3 method for class 'gplm'
fitted(object, ...)
```

Arguments

formula	a formula object describing the model to be estimated.
data	a data.frame or matrix holding a panel data set. If no index variables are provided, the panel must be balanced and ordered in the long format $\boldsymbol{Y} = (Y_1', \ldots, Y_N')'$, $Y_i = (Y_{i1}, \ldots, Y_{iT})'$ with $Y_{it} = (y_{it}, x_{it}')'$. Conversely, if data is not ordered or not balanced, data must include two index variables that declare the cross-sectional unit i and the time period t of each observation.
groups	a numerical or character vector of length N that indicates the group membership of each cross-sectional unit i.
index	a character vector holding two strings. The first string denotes the name of the index variable identifying the cross-sectional unit i and the second string represents the name of the variable declaring the time period t. The data is automatically sorted according to the variables in index, which may produce errors when the time index is a character variable. In case of a balanced panel data set that is ordered in the long format, index can be left empty if the the number of time periods n_periods is supplied.
n_periods	the number of observed time periods T. If an index is passed, this argument can be left empty.
method	the estimation method. Options are "PLS" for using the penalized least squares (*PLS*) algorithm. We recommend *PLS* in case of (weakly) exogenous regressors (Mehrabani, 2023, sec. 2.2). "PGMM" for using the penalized Generalized Method of Moments (*PGMM*). *PGMM* is required when instrumenting endogenous regressors, in which case a matrix $\boldsymbol{Z}$ containing the necessary exogenous instruments must be supplied (Mehrabani, 2023, sec. 2.3). Default is "PLS".
Z	a $NT \times q$ matrix or data.frame of exogenous instruments, where $q \geq p$, $\boldsymbol{Z} = (z_1, \ldots, z_N)'$, $z_i = (z_{i1}, \ldots, z_{iT})'$ and z_{it} is a $q \times 1$ vector. Z is only required when method = "PGMM" is selected. When using "PLS", the argument can be left empty or it is disregarded. Default is NULL.
bias_correc	logical. If TRUE, a Split-panel Jackknife bias correction following Dhaene and Jochmans (2015) is applied to the slope parameters. We recommend using the correction when working with dynamic panels. Default is FALSE.

rho	a tuning parameter balancing the fitness and penalty terms in the IC. If left unspecified, the heuristic $\rho = 0.07\frac{\log(NT)}{\sqrt{NT}}$ of Mehrabani (2023, sec. 6) is used. We recommend the default.
verbose	logical. If TRUE, helpful warning messages are shown. Default is TRUE.
parallel	logical. If TRUE, certain operations are parallelized across multiple cores. Default is TRUE.
...	ellipsis
x	of class gplm.
object	of class gplm.

Details

Consider the grouped panel data model

$$y_{it} = \gamma_i + \beta_i' x_{it} + \epsilon_{it}, \quad i = 1, \ldots, N,\ t = 1, \ldots, T,$$

where y_{it} is the scalar dependent variable, γ_i is an individual fixed effect, x_{it} is a $p \times 1$ vector of explanatory variables, and ϵ_{it} is a zero mean error. The coefficient vector β_i is subject to the observed group pattern

$$\beta_i = \sum_{k=1}^{K} \alpha_k \mathbf{1}\{i \in G_k\},$$

with $\cup_{k=1}^{K} G_k = \{1, \ldots, N\}$, $G_k \cap G_j = \emptyset$ and $\|\alpha_k - \alpha_j\| \neq 0$ for any $k \neq j$, $k = 1, \ldots, K$.

Using *PLS*, the group-specific coefficients for group k are obtained via *OLS*

$$\hat{\alpha}_k = \left(\sum_{i \in G_k} \sum_{t=1}^{T} \tilde{x}_{it} \tilde{x}_{it}' \right)^{-1} \sum_{i \in G_k} \sum_{t=1}^{T} \tilde{x}_{it} \tilde{y}_{it},$$

where $\tilde{a}_{it} = a_{it} - T^{-1} \sum_{t=1}^{T} a_{it}$, $a = \{y, x\}$ to concentrate out the individual fixed effects γ_i (within-transformation).

In case of *PGMM*, the slope coefficients are derived as

$$\hat{\alpha}_k = \left(\left[\sum_{i \in G_k} T^{-1} \sum_{t=1}^{T} z_{it} \Delta x_{it} \right]' W_k \left[\sum_{i \in G_k} T^{-1} \sum_{t=1}^{T} z_{it} \Delta x_{it} \right] \right)^{-1}$$

$$\left[\sum_{i \in G_k} T^{-1} \sum_{t=1}^{T} z_{it} \Delta x_{it} \right]' W_k \left[\sum_{i \in G_k} T^{-1} \sum_{t=1}^{T} z_{it} \Delta y_{it} \right],$$

where W_k is a $q \times q$ p.d. symmetric weight matrix and Δ denotes the first difference operator $\Delta x_{it} = x_{it} - x_{it-1}$ (first-difference transformation).

Value

An object of class `gplm` holding

`model`	a `data.frame` containing the dependent and explanatory variables as well as cross-sectional and time indices,
`coefficients`	a $K \times p$ matrix of the group-specific parameter estimates,
`groups`	a `list` containing (i) the total number of groups K and (ii) a vector of group memberships $g_1, \ldots, g_N$), where $g_i = k$ if i is assigned to group k,
`residuals`	a vector of residuals of the demeaned model,
`fitted`	a vector of fitted values of the demeaned model,
`args`	a `list` of additional arguments,
`IC`	a `list` containing (i) the value of the IC and (ii) the *MSE*,
`call`	the function call.

A `gplm` object has `print`, `summary`, `fitted`, `residuals`, `formula`, `df.residual`, and `coef` S3 methods.

Author(s)

Paul Haimerl

References

Dhaene, G., & Jochmans, K. (2015). Split-panel jackknife estimation of fixed-effect models. *The Review of Economic Studies*, 82(3), 991-1030. doi:10.1093/restud/rdv007. Mehrabani, A. (2023). Estimation and identification of latent group structures in panel data. *Journal of Econometrics*, 235(2), 1464-1482. doi:10.1016/j.jeconom.2022.12.002.

Examples

```
# Simulate a panel with a group structure
sim <- sim_DGP(N = 20, n_periods = 80, p = 2, n_groups = 3)
y <- sim$y
X <- sim$X
groups <- sim$groups
df <- cbind(y = c(y), X)

# Estimate the grouped panel data model
estim <- grouped_plm(y ~ ., data = df, groups = groups, n_periods = 80, method = "PLS")
summary(estim)

# Lets pass a panel data set with explicit cross-sectional and time indicators
i_index <- rep(1:20, each = 80)
t_index <- rep(1:80, 20)
df <- data.frame(y = c(y), X, i_index = i_index, t_index = t_index)
estim <- grouped_plm(
  y ~ .,
  data = df, index = c("i_index", "t_index"), groups = groups, method = "PLS"
)
summary(estim)
```

grouped_tv_plm *Grouped Time-varying Panel Data Model*

Description

Estimate a grouped time-varying panel data model given an observed group structure. Coefficient functions are homogeneous within groups but heterogeneous across groups. The time-varying coefficients are modeled as polynomial B-splines. The function supports both static and dynamic panel data models.

Usage

```
grouped_tv_plm(
  formula,
  data,
  groups,
  index = NULL,
  n_periods = NULL,
  d = 3,
  M = floor(length(y)^(1/7) - log(p)),
  const_coef = NULL,
  rho = 0.04 * log(N * n_periods)/sqrt(N * n_periods),
  verbose = TRUE,
  parallel = TRUE,
  ...
)

## S3 method for class 'tv_gplm'
summary(object, ...)

## S3 method for class 'tv_gplm'
formula(x, ...)

## S3 method for class 'tv_gplm'
df.residual(object, ...)

## S3 method for class 'tv_gplm'
print(x, ...)

## S3 method for class 'tv_gplm'
coef(object, ...)

## S3 method for class 'tv_gplm'
residuals(object, ...)

## S3 method for class 'tv_gplm'
fitted(object, ...)
```

Arguments

formula	a formula object describing the model to be estimated.
data	a data.frame or matrix holding a panel data set. If no index variables are provided, the panel must be balanced and ordered in the long format $\boldsymbol{Y} = (Y_1', \ldots, Y_N')'$, $Y_i = (Y_{i1}, \ldots, Y_{iT})'$ with $Y_{it} = (y_{it}, x_{it}')'$. Conversely, if data is not ordered or not balanced, data must include two index variables that declare the cross-sectional unit i and the time period t of each observation.
groups	a numerical or character vector of length N that indicates the group membership of each cross-sectional unit i.
index	a character vector holding two strings. The first string denotes the name of the index variable identifying the cross-sectional unit i, and the second string represents the name of the variable declaring the time period t. The data is automatically sorted according to the variables in index, which may produce errors when the time index is a character variable. In case of a balanced panel data set that is ordered in the long format, index can be left empty if the the number of time periods n_periods is supplied.
n_periods	the number of observed time periods T. If an index character vector is passed, this argument can be left empty. Default is Null.
d	the polynomial degree of the B-splines. Default is 3.
M	the number of interior knots of the B-splines. If left unspecified, the default heuristic $M = \text{floor}((NT)^{\frac{1}{7}} - \log(p))$ is used. Note that M does not include the boundary knots and the entire sequence of knots is of length $M + d + 1$.
const_coef	a character vector containing the variable names of explanatory variables that enter with time-constant coefficients.
rho	the tuning parameter balancing the fitness and penalty terms in the IC. If left unspecified, the heuristic $\rho = 0.07\frac{\log(NT)}{\sqrt{NT}}$ of Mehrabani (2023, sec. 6) is used. We recommend the default.
verbose	logical. If TRUE, helpful warning messages are shown. Default is TRUE.
parallel	logical. If TRUE, certain operations are parallelized across multiple cores. Default is TRUE.
...	ellipsis
object	of class tv_gplm.
x	of class tv_gplm.

Details

Consider the grouped time-varying panel data model

$$y_{it} = \gamma_i + \beta_i'(t/T)x_{it} + \epsilon_{it}, \quad i = 1, \ldots, N,\ t = 1, \ldots, T,$$

where y_{it} is the scalar dependent variable, γ_i is an individual fixed effect, x_{it} is a $p \times 1$ vector of explanatory variables, and ϵ_{it} is a zero mean error. The coefficient vector $\beta_i(t/T)$ is subject to the observed group pattern

$$\beta_i\left(\frac{t}{T}\right) = \sum_{k=1}^{K} \alpha_k\left(\frac{t}{T}\right)\mathbf{1}\{i \in G_k\},$$

with $\cup_{k=1}^{K} G_k = \{1, \dots, N\}$, $G_k \cap G_j = \emptyset$ and $\|\alpha_k - \alpha_j\| \neq 0$ for any $k \neq j$, $k = 1, \dots, K$.

$\alpha_k(t/T)$ and, in turn, $\beta_i(t/T)$ is estimated as polynomial B-splines using the penalized sieve-technique. To this end, let $B(v)$ denote a $M + d + 1$ vector of polynomial spline basis functions, where d represents the polynomial degree and M gives the number of interior knots of the B-spline. $\alpha_k(t/T)$ is approximated by forming a linear combination of the basis functions $\alpha_k(t/T) \approx \xi_k' B(t/T)$, where ξ_k is a $(M + d + 1) \times p$ coefficient matrix.

The explanatory variables are projected onto the spline basis system, which results in the $(M + d + 1)p \times 1$ vector $z_{it} = x_{it} \otimes B(v)$. Subsequently, the DGP can be reformulated as

$$y_{it} = \gamma_i + z_{it}' \text{vec}(\pi_i) + u_{it},$$

where $\pi_i = \xi_k$ if $i \in G_k$, $u_{it} = \epsilon_{it} + \eta_{it}$, and η_{it} reflects a sieve approximation error. We refer to Su et al. (2019, sec. 2) for more details on the sieve technique.

Finally, $\hat{\alpha}_k(t/T)$ is obtained as $\hat{\alpha}_k(t/T) = \hat{\xi}_k' B(t/T)$, where the vector of control points ξ_k is estimated using *OLS*

$$\hat{\xi}_k = \left(\sum_{i \in G_k} \sum_{t=1}^{T} \tilde{z}_{it} \tilde{z}_{it}' \right)^{-1} \sum_{i \in G_k} \sum_{t=1}^{T} \tilde{z}_{it} \tilde{y}_{it},$$

and $\tilde{a}_{it} = a_{it} - T^{-1} \sum_{t=1}^{T} a_{it}$, $a = \{y, z\}$ to concentrate out the fixed effect γ_i (within-transformation).

In case of an unbalanced panel data set, the earliest and latest available observations per group define the start and end-points of the interval on which the group-specific time-varying coefficients are defined.

Value

An object of class `tv_gplm` holding

`model`	a `data.frame` containing the dependent and explanatory variables as well as cross-sectional and time indices,
`coefficients`	let $p^{(1)}$ denote the number of time-varying and $p^{(2)}$ the number of time constant coefficients. A `list` holding (i) a $T \times p^{(1)} \times K$ array of the group-specific functional coefficients and (ii) a $K \times p^{(2)}$ matrix of time-constant estimates.
`groups`	a `list` containing (i) the total number of groups K and (ii) a vector of group memberships $(\hat{g}_1, \dots, \hat{g}_N)$, where $\hat{g}_i = k$ if i is part of group k,
`residuals`	a vector of residuals of the demeaned model,
`fitted`	a vector of fitted values of the demeaned model,
`args`	a `list` of additional arguments,
`IC`	a `list` containing (i) the value of the IC and (ii) the *MSE*,
`call`	the function call.

An object of class `tv_gplm` has `print`, `summary`, `fitted`, `residuals`, `formula`, `df.residual` and `coef` S3 methods.

Author(s)

Paul Haimerl

References

Su, L., Wang, X., & Jin, S. (2019). Sieve estimation of time-varying panel data models with latent structures. *Journal of Business & Economic Statistics*, 37(2), 334-349. doi:10.1080/07350015.2017.1340299.

Examples

```
# Simulate a time-varying panel with a trend and a group pattern
set.seed(1)
sim <- sim_tv_DGP(N = 10, n_periods = 50, intercept = TRUE, p = 2)
df <- data.frame(y = c(sim$y))
groups <- sim$groups

# Estimate the time-varying grouped panel data model
estim <- grouped_tv_plm(y ~ ., data = df, n_periods = 50, groups = groups)
summary(estim)
```

---

pagfl *Pairwise Adaptive Group Fused Lasso*

Description

Estimate panel data models with a latent group structure using the pairwise adaptive group fused Lasso (*PAGFL*) by Mehrabani (2023). The *PAGFL* jointly identifies the group structure and group-specific slope parameters. The function supports both static and dynamic panels, with or without endogenous regressors.

Usage

```
pagfl(
  formula,
  data,
  index = NULL,
  n_periods = NULL,
  lambda,
  method = "PLS",
  Z = NULL,
  min_group_frac = 0.05,
  bias_correc = FALSE,
  kappa = 2,
  max_iter = 5000,
  tol_convergence = 1e-08,
  tol_group = 0.001,
  rho = 0.07 * log(N * n_periods)/sqrt(N * n_periods),
  varrho = max(sqrt(5 * N * n_periods * p)/log(N * n_periods * p) - 7, 1),
  verbose = TRUE,
  parallel = TRUE,
```

```
  ...
)

## S3 method for class 'pagfl'
print(x, ...)

## S3 method for class 'pagfl'
formula(x, ...)

## S3 method for class 'pagfl'
df.residual(object, ...)

## S3 method for class 'pagfl'
summary(object, ...)

## S3 method for class 'pagfl'
coef(object, ...)

## S3 method for class 'pagfl'
residuals(object, ...)

## S3 method for class 'pagfl'
fitted(object, ...)
```

Arguments

formula	a formula object describing the model to be estimated.
data	a data.frame or matrix holding a panel data set. If no index variables are provided, the panel must be balanced and ordered in the long format $\boldsymbol{Y} = (Y_1', \dots, Y_N')'$, $Y_i = (Y_{i1}, \dots, Y_{iT})'$ with $Y_{it} = (y_{it}, x_{it}')'$. Conversely, if data is not ordered or not balanced, data must include two index variables that declare the cross-sectional unit i and the time period t of each observation.
index	a character vector holding two strings. The first string denotes the name of the index variable identifying the cross-sectional unit i and the second string represents the name of the variable declaring the time period t. The data is automatically sorted according to the variables in index, which may produce errors when the time index is a character variable. In case of a balanced panel data set that is ordered in the long format, index can be left empty if the the number of time periods n_periods is supplied.
n_periods	the number of observed time periods T. If an index character vector is passed, this argument can be left empty.
lambda	the tuning parameter determining the strength of the penalty term. Either a single λ or a vector of candidate values can be passed. If a vector is supplied, a BIC-type IC automatically selects the best fitting λ value.
method	the estimation method. Options are
	"PLS" for using the penalized least squares (*PLS*) algorithm. We recommend *PLS* in case of (weakly) exogenous regressors (Mehrabani, 2023, sec. 2.2).

"PGMM" for using the penalized Generalized Method of Moments (*PGMM*). *PGMM* is required when instrumenting endogenous regressors, in which case a matrix $\boldsymbol{Z}$ containing the necessary exogenous instruments must be supplied (Mehrabani, 2023, sec. 2.3).

Default is "PLS".

`Z` a $NT \times q$ `matrix` or `data.frame` of exogenous instruments, where $q \geq p$, $\boldsymbol{Z} = (z_1, \ldots, z_N)'$, $z_i = (z_{i1}, \ldots, z_{iT})'$ and z_{it} is a $q \times 1$ vector. `Z` is only required when `method = "PGMM"` is selected. When using "PLS", either pass `NULL` or `Z` is disregarded. Default is `NULL`.

`min_group_frac` the minimum group cardinality as a fraction of the total number of individuals N. In case a group falls short of this threshold, each of its members is allocated to one of the remaining groups according to the *MSE*. Default is 0.05.

`bias_correc` logical. If `TRUE`, a Split-panel Jackknife bias correction following Dhaene and Jochmans (2015) is applied to the slope parameters. We recommend using the correction when working with dynamic panels. Default is `FALSE`.

`kappa` the a non-negative weight used to obtain the adaptive penalty weights. Default is 2.

`max_iter` the maximum number of iterations for the *ADMM* estimation algorithm. Default is $1 * 10^4$.

`tol_convergence`
the tolerance limit for the stopping criterion of the iterative *ADMM* estimation algorithm. Default is $1 * 10^{-8}$.

`tol_group` the tolerance limit for within-group differences. Two individuals i, j are assigned to the same group if the Frobenius norm of their coefficient vector difference is below this threshold. Default is $1 * 10^{-3}$.

`rho` the tuning parameter balancing the fitness and penalty terms in the IC that determines the penalty parameter λ. If left unspecified, the heuristic $\rho = 0.07 \frac{\log(NT)}{\sqrt{NT}}$ of Mehrabani (2023, sec. 6) is used. We recommend the default.

`varrho` the non-negative Lagrangian *ADMM* penalty parameter. For *PLS*, the ϱ value is trivial. However, for *PGMM*, small values lead to slow convergence. If left unspecified, the default heuristic $\varrho = \max(\frac{\sqrt{5NTp}}{\log(NTp)} - 7, 1)$ is used.

`verbose` logical. If `TRUE`, helpful warning messages are shown. Default is `TRUE`.

`parallel` logical. If `TRUE`, certain operations are parallelized across multiple cores. Default is `TRUE`.

`...` ellipsis

`x` of class `pagfl`.

`object` of class `pagfl`.

Details

Consider the grouped panel data model

$$y_{it} = \gamma_i + \beta_i' x_{it} + \epsilon_{it}, \quad i = 1, \ldots, N, \; t = 1, \ldots, T,$$

where y_{it} is the scalar dependent variable, γ_i is an individual fixed effect, x_{it} is a $p \times 1$ vector of weakly exogenous explanatory variables, and ϵ_{it} is a zero mean error. The coefficient vector β_i is subject to the latent group pattern

$$\beta_i = \sum_{k=1}^{K} \alpha_k \mathbf{1}\{i \in G_k\},$$

with $\cup_{k=1}^{K} G_k = \{1, \ldots, N\}$, $G_k \cap G_j = \emptyset$ and $\|\alpha_k - \alpha_j\| \neq 0$ for any $k \neq j$, $k = 1, \ldots, K$.

The *PLS* method jointly estimates the latent group structure and group-specific coefficients by minimizing the criterion

$$Q_{NT}(\boldsymbol{\beta}, \lambda) = \frac{1}{T} \sum_{i=1}^{N} \sum_{t=1}^{T} (\tilde{y}_{it} - \beta_i' \tilde{x}_{it})^2 + \frac{\lambda}{N} \sum_{i=1}^{N-1} \sum_{j>i}^{N} \dot{\omega}_{ij} \|\beta_i - \beta_j\|$$

with respect to $\boldsymbol{\beta} = (\beta_1', \ldots, \beta_N')'$. $\tilde{a}_{it} = a_{it} - T^{-1} \sum_{t=1}^{T} a_{it}$, $a = \{y, x\}$ to concentrate out the individual fixed effects γ_i. λ is the penalty tuning parameter and $\dot{\omega}_{ij}$ reflects adaptive penalty weights (see Mehrabani, 2023, eq. 2.6). $\|\cdot\|$ denotes the Frobenius norm. The adaptive weights $\dot{w}_{ij}$ are obtained by a preliminary individual least squares estimation. The criterion function is minimized via an iterative alternating direction method of multipliers (*ADMM*) algorithm (see Mehrabani, 2023, sec. 5.1).

PGMM employs a set of instruments $\boldsymbol{Z}$ to control for endogenous regressors. Using *PGMM*, $\boldsymbol{\beta}$ is estimated by minimizing

$$Q_{NT}(\boldsymbol{\beta}, \lambda) = \sum_{i=1}^{N} \left[\frac{1}{N} \sum_{t=1}^{T} z_{it} (\Delta y_{it} - \beta_i' \Delta x_{it}) \right]' W_i \left[\frac{1}{T} \sum_{t=1}^{T} z_{it} (\Delta y_{it} - \beta_i' \Delta x_{it}) \right]$$

$$+ \frac{\lambda}{N} \sum_{i=1}^{N-1} \sum_{j>i}^{N} \ddot{\omega}_{ij} \|\beta_i - \beta_j\|.$$

$\ddot{\omega}_{ij}$ are obtained by an initial *GMM* estimation. Δ gives the first differences operator $\Delta y_{it} = y_{it} - y_{it-1}$. W_i represents a data-driven $q \times q$ weight matrix. I refer to Mehrabani (2023, eq. 2.10) for more details. Again, the criterion function is minimized using an efficient *ADMM* algorithm (Mehrabani, 2023, sec. 5.2).

Two individuals are assigned to the same group if $\|\hat{\beta}_i - \hat{\beta}_j\| \leq \epsilon_{\text{tol}}$, where ϵ_{tol} is determined by `tol_group`. Subsequently, the number of groups follows as the number of distinct elements in $\hat{\boldsymbol{\beta}}$. Given an estimated group structure, it is straightforward to obtain post-Lasso estimates using group-wise least squares or *GMM* (see `grouped_plm`).

We recommend identifying a suitable λ parameter by passing a logarithmically spaced grid of candidate values with a lower limit close to 0 and an upper limit that leads to a fully homogeneous panel. A BIC-type information criterion then selects the best fitting λ value.

Value

An object of class `pagfl` holding

`model` a `data.frame` containing the dependent and explanatory variables as well as cross-sectional and time indices,

`coefficients`	a $\hat{K} \times p$ matrix of the post-Lasso group-specific parameter estimates,
`groups`	a `list` containing (i) the total number of groups $\hat{K}$ and (ii) a vector of estimated group memberships $(\hat{g}_1, \ldots, \hat{g}_N)$, where $\hat{g}_i = k$ if i is assigned to group k,
`residuals`	a vector of residuals of the demeaned model,
`fitted`	a vector of fitted values of the demeaned model,
`args`	a `list` of additional arguments,
`IC`	a `list` containing (i) the value of the IC, (ii) the employed tuning parameter λ, and (iii) the *MSE*,
`convergence`	a `list` containing (i) a logical variable indicating if convergence was achieved and (ii) the number of executed *ADMM* algorithm iterations,
`call`	the function call.

A `pagfl` object has `print`, `summary`, `fitted`, `residuals`, `formula`, `df.residual`, and `coef` S3 methods.

Author(s)

Paul Haimerl

References

Dhaene, G., & Jochmans, K. (2015). Split-panel jackknife estimation of fixed-effect models. *The Review of Economic Studies*, 82(3), 991-1030. doi:10.1093/restud/rdv007. Mehrabani, A. (2023). Estimation and identification of latent group structures in panel data. *Journal of Econometrics*, 235(2), 1464-1482. doi:10.1016/j.jeconom.2022.12.002.

Examples

```
# Simulate a panel with a group structure
sim <- sim_DGP(N = 20, n_periods = 80, p = 2, n_groups = 3)
y <- sim$y
X <- sim$X
df <- cbind(y = c(y), X)

# Run the PAGFL procedure
estim <- pagfl(y ~ ., data = df, n_periods = 80, lambda = 0.5, method = "PLS")
summary(estim)

# Lets pass a panel data set with explicit cross-sectional and time indicators
i_index <- rep(1:20, each = 80)
t_index <- rep(1:80, 20)
df <- data.frame(y = c(y), X, i_index = i_index, t_index = t_index)
estim <- pagfl(
  y ~ ., data = df, index = c("i_index", "t_index"), lambda = 0.5, method = "PLS"
)
summary(estim)
```

`sim_DGP` *Simulate a Panel With a Group Structure in the Slope Coefficients*

Description

Construct a static or dynamic, exogenous or endogenous panel data set subject to a group structure in the slope coefficients with optional $AR(1)$ or $GARCH(1,1)$ innovations.

Usage

```
sim_DGP(
  N = 50,
  n_periods = 40,
  p = 2,
  n_groups = 3,
  group_proportions = NULL,
  error_spec = "iid",
  dynamic = FALSE,
  dyn_panel = lifecycle::deprecated(),
  q = NULL,
  alpha_0 = NULL
)
```

Arguments

`N`	the number of cross-sectional units. Default is 50.
`n_periods`	the number of simulated time periods T. Default is 40.
`p`	the number of explanatory variables. Default is 2.
`n_groups`	the number of groups K. Default is 3.
`group_proportions`	a numeric vector of length `n_groups` indicating size of each group as a fraction of N. If `NULL`, all groups are of size N/K. Default is `NULL`.
`error_spec`	options include `"iid"` for iid errors. `"AR"` for an $AR(1)$ error process with an autoregressive coefficient of 0.5. `"GARCH"` for a $GARCH(1,1)$ error process with a 0.05 constant, a 0.05 ARCH and a 0.9 GARCH coefficient. Default is `"iid"`.
`dynamic`	Logical. If `TRUE`, the panel includes one stationary autoregressive lag of y_{it} as an explanatory variable (see sec. Details for more information on the AR coefficient). Default is `FALSE`.
`dyn_panel`	**[Deprecated]** deprecated and replaced by `dynamic`.

q — the number of exogenous instruments when a panel with endogenous regressors is to be simulated. If panel data set with exogenous regressors is supposed to be generated, pass NULL. Default is NULL.

alpha_0 — a $K \times p$ matrix of group-specific coefficients. If dynamic = TRUE, the first column represents the stationary AR coefficient. If NULL, the coefficients are drawn randomly (see sec. Details). Default is NULL.

Details

The scalar dependent variable y_{it} is generated according to the following grouped panel data model

$$y_{it} = \gamma_i + \beta_i' x_{it} + u_{it}, \quad i = \{1, \dots, N\}, \quad t = \{1, \dots, T\}.$$

γ_i represents individual fixed effects and x_{it} a $p \times 1$ vector of regressors. The individual slope coefficient vectors β_i are subject to a group structure

$$\beta_i = \sum_{k=1}^{K} \alpha_k \mathbf{1}\{i \in G_k\},$$

with $\cup_{k=1}^{K} G_k = \{1, \dots, N\}$, $G_k \cap G_j = \emptyset$ and $\|\alpha_k - \alpha_j\| \neq 0$ for any $k \neq j$, $k = 1, \dots, K$. The total number of groups K is determined by n_groups.

If a panel data set with exogenous regressors is generated (set q = NULL), the explanatory variables are simulated according to

$$x_{it,j} = 0.2\gamma_i + e_{it,j}, \quad \gamma_i, e_{it,j} \sim i.i.d.N(0,1), \quad j = \{1, \dots, p\},$$

where $e_{it,j}$ denotes a series of innovations. γ_i and e_i are independent of each other.

In case alpha_0 = NULL, the group-level slope parameters α_k are drawn from $\sim U[-2, 2]$.

If a dynamic panel is specified (dynamic = TRUE), the AR coefficients β_i^{AR} are drawn from a uniform distribution with support $(-1, 1)$ and $x_{it,j} = e_{it,j}$. Moreover, the individual fixed effects enter the dependent variable via $(1 - \beta_i^{\text{AR}})\gamma_i$ to account for the autoregressive dependency. We refer to Mehrabani (2023, sec 6) for details.

When specifying an endogenous panel (set q to $q \geq p$), the $e_{it,j}$ correlate with the cross-sectional innovations u_{it} by a magnitude of 0.5 to produce endogenous regressors ($\text{E}(u|X) \neq 0$). However, the endogenous regressors can be accounted for by exploiting the q instruments in $\boldsymbol{Z}$, for which $\text{E}(u|Z) = 0$ holds. The instruments and the first stage coefficients are generated in the same fashion as $\boldsymbol{X}$ and $\boldsymbol{\alpha}$ when q = NULL.

The function nests, among other, the DGPs employed in the simulation study of Mehrabani (2023, sec. 6).

Value

A list holding

alpha — the $K \times p$ matrix of group-specific slope parameters. If dynamic = TRUE, the first column holds the AR coefficient.

groups — a vector indicating the group memberships $(g_1, \dots, g_N)$, where $g_i = k$ if $i \in$ group k.

y	a $NT \times 1$ vector of the dependent variable, with $\boldsymbol{y} = (y_1, \ldots, y_N)'$, $y_i = (y_{i1}, \ldots, y_{iT})'$ and the scalar y_{it}.
X	a $NT \times p$ matrix of explanatory variables, with $\boldsymbol{X} = (x_1, \ldots, x_N)'$, $x_i = (x_{i1}, \ldots, x_{iT})'$ and the $p \times 1$ vector x_{it}.
Z	a $NT \times q$ matrix of instruments , where $q \geq p$, $\boldsymbol{Z} = (z_1, \ldots, z_N)'$, $z_i = (z_{i1}, \ldots, z_{iT})'$ and z_{it} is a $q \times 1$ vector. In case a panel with exogenous regressors is generated (q = NULL), $\boldsymbol{Z}$ equals NULL.
data	a $NT \times (p+1)$ data.frame of the outcome and the explanatory variables.

Author(s)

Paul Haimerl

References

Mehrabani, A. (2023). Estimation and identification of latent group structures in panel data. *Journal of Econometrics*, 235(2), 1464-1482. doi:10.1016/j.jeconom.2022.12.002.

Examples

```
# Simulate DGP 1 from Mehrabani (2023, sec. 6)
alpha_0_DGP1 <- matrix(c(0.4, 1, 1.6, 1.6, 1, 0.4), ncol = 2)
DGP1 <- sim_DGP(
  N = 50, n_periods = 20, p = 2, n_groups = 3,
  group_proportions = c(.4, .3, .3), alpha_0 = alpha_0_DGP1
)
```

---

sim_tv_DGP *Simulate a Time-varying Panel With a Group Structure in the Slope Coefficients*

Description

Construct a time-varying panel data set subject to a group structure in the slope coefficients with optional $AR(1)$ innovations.

Usage

```
sim_tv_DGP(
  N = 50,
  n_periods = 40,
  intercept = TRUE,
  p = 1,
  n_groups = 3,
  d = 3,
  dynamic = FALSE,
  group_proportions = NULL,
```

```
  error_spec = "iid",
  locations = NULL,
  scales = NULL,
  polynomial_coef = NULL,
  sd_error = 1,
  DGP = lifecycle::deprecated()
)
```

Arguments

N	the number of cross-sectional units. Default is 50.
n_periods	the number of simulated time periods T. Default is 40.
intercept	logical. If TRUE, a time-varying intercept is generated.
p	the number of simulated explanatory variables
n_groups	the number of groups K. Default is 3.
d	the polynomial degree used to construct the time-varying coefficients.
dynamic	Logical. If TRUE, the panel includes one stationary autoregressive lag of y_{it} as a regressor. Default is FALSE.
group_proportions	a numeric vector of length n_groups indicating size of each group as a fraction of N. If NULL, all groups are of size N/K. Default is NULL.
error_spec	options include "iid" for iid errors. "AR" for an $AR(1)$ error process with an autoregressive coefficient of 0.5. Default is "iid".
locations	a $p \times K$ matrix of location parameters of a logistic distribution function used to construct the time-varying coefficients. If left empty, the location parameters are drawn randomly. Default is NULL.
scales	a $p \times K$ matrix of scale parameters of a logistic distribution function used to construct the time-varying coefficients. If left empty, the location parameters are drawn randomly. Default is NULL.
polynomial_coef	a $p \times d \times K$ array of coefficients for a the polynomials used to construct the time-varying coefficients. If left empty, the location parameters are drawn randomly. Default is NULL.
sd_error	standard deviation of the cross-sectional errors. Default is 1.
DGP	**[Deprecated]** the data generating process. Options are **1** generates a trend only. **2** simulates a trend and an additional exogenous explanatory variable. **1** draws a dynamic panel data model with one AR lag.

Details

The scalar dependent variable y_{it} is generated according to the following time-varying grouped panel data model

$$y_{it} = \gamma_i + \beta_{it}' x_{it} + u_{it}, \quad i = 1, \ldots, N, \; t = 1, \ldots, T,$$

where γ_i is an individual fixed effect and x_{it} is a $p \times 1$ vector of explanatory variables. The coefficient vector $\beta_i = \{\beta_{i1}', \ldots, \beta_{iT}'\}'$ is subject to the group pattern

$$\beta_i \left(\frac{t}{T} \right) = \sum_{k=1}^{K} \alpha_k \left(\frac{t}{T} \right) \mathbf{1}\{i \in G_k\},$$

with $\cup_{k=1}^{K} G_k = \{1, \ldots, N\}$, $G_k \cap G_j = \emptyset$ and $\sup_{v \in [0,1]} (\|\alpha_k(v) - \alpha_j(v)\|) \neq 0$ for any $k \neq j$, $k = 1, \ldots, K$. The total number of groups K is determined by `n_groups`.

The predictors are simulated as:

$$x_{it,j} = 0.2\gamma_i + e_{it,j}, \quad \gamma_i, e_{it,j} \sim i.i.d.N(0,1), \quad j = \{1, \ldots, p\},$$

where $e_{it,j}$ denotes a series of innovations. γ_i and e_i are independent of each other.

The errors u_{it} feature a iid standard normal distribution.

In case `locations = NULL`, the location parameters are drawn from $\sim U[0.3, 0.9]$. In case `scales = NULL`, the scale parameters are drawn from $\sim U[0.01, 0.09]$. In case `polynomial_coef = NULL`, the polynomial coefficients are drawn from $\sim U[-20, 20]$ and normalized so that all coefficients of one polynomial sum up to 1. The final coefficient function follows as $\alpha_k(t/T) = 3 * F(t/T, location, scale) + \sum_{j=1}^{d} a_j (t/T)^j$, where $F(\cdot, location, scale)$ denotes a cumulative logistic distribution function and a_j reflects a polynomial coefficient.

Value

A list holding

`alpha`	a $T \times p \times K$ array of group-specific time-varying parameters
`beta`	a $T \times p \times N$ array of individual time-varying parameters
`groups`	a vector indicating the group memberships $(g_1, \ldots, g_N)$, where $g_i = k$ if $i \in$ group k.
`y`	a $NT \times 1$ vector of the dependent variable, with $\boldsymbol{y} = (y_1, \ldots, y_N)'$, $y_i = (y_{i1}, \ldots, y_{iT})'$ and the scalar y_{it}.
`X`	a $NT \times p$ matrix of explanatory variables, with $\boldsymbol{X} = (x_1, \ldots, x_N)'$, $x_i = (x_{i1}, \ldots, x_{iT})'$ and the $p \times 1$ vector x_{it}.
`data`	a $NT \times (p+1)$ data.frame of the outcome and the explanatory variables.

Author(s)

Paul Haimerl

Examples

```
# Simulate a time-varying panel subject to a time trend and a group structure
sim <- sim_tv_DGP(N = 20, n_periods = 50, intercept = TRUE, p = 1)
y <- sim$y
```

---

tv_pagfl *Time-varying Pairwise Adaptive Group Fused Lasso*

Description

Estimate a time-varying panel data model with a latent group structure using the pairwise adaptive group fused lasso (*time-varying PAGFL*). The *time-varying PAGFL* jointly identifies the latent group structure and group-specific time-varying functional coefficients. The time-varying coefficients are modeled as polynomial B-splines. The function supports both static and dynamic panel data models.

Usage

```
tv_pagfl(
  formula,
  data,
  index = NULL,
  n_periods = NULL,
  lambda,
  d = 3,
  M = floor(length(y)^(1/7) - log(p)),
  min_group_frac = 0.05,
  const_coef = NULL,
  kappa = 2,
  max_iter = 50000,
  tol_convergence = 1e-10,
  tol_group = 0.001,
  rho = 0.04 * log(N * n_periods)/sqrt(N * n_periods),
  varrho = 1,
  verbose = TRUE,
  parallel = TRUE,
  ...
)

## S3 method for class 'tvpagfl'
summary(object, ...)

## S3 method for class 'tvpagfl'
formula(x, ...)

## S3 method for class 'tvpagfl'
df.residual(object, ...)
```

```
## S3 method for class 'tvpagfl'
print(x, ...)

## S3 method for class 'tvpagfl'
coef(object, ...)

## S3 method for class 'tvpagfl'
residuals(object, ...)

## S3 method for class 'tvpagfl'
fitted(object, ...)
```

Arguments

formula	a formula object describing the model to be estimated.
data	a data.frame or matrix holding a panel data set. If no index variables are provided, the panel must be balanced and ordered in the long format $\boldsymbol{Y} = (Y_1', \dots, Y_N')'$, $Y_i = (Y_{i1}, \dots, Y_{iT})'$ with $Y_{it} = (y_{it}, x_{it}')'$. Conversely, if data is not ordered or not balanced, data must include two index variables that declare the cross-sectional unit i and the time period t of each observation.
index	a character vector holding two strings. The first string denotes the name of the index variable identifying the cross-sectional unit i and the second string represents the name of the variable declaring the time period t. The data is automatically sorted according to the variables in index, which may produce errors when the time index is a character variable. In case of a balanced panel data set that is ordered in the long format, index can be left empty if the the number of time periods n_periods is supplied.
n_periods	the number of observed time periods T. If an index character vector is passed, this argument can be left empty. Default is Null.
lambda	the tuning parameter determining the strength of the penalty term. Either a single λ or a vector of candidate values can be passed. If a vector is supplied, a BIC-type IC automatically selects the best fitting λ value.
d	the polynomial degree of the B-splines. Default is 3.
M	the number of interior knots of the B-splines. If left unspecified, the default heuristic $M = \text{floor}((NT)^{\frac{1}{7}} - \log(p))$ is used. Note that M does not include the boundary knots and the entire sequence of knots is of length $M + d + 1$.
min_group_frac	the minimum group cardinality as a fraction of the total number of individuals N. In case a group falls short of this threshold, each of its members is allocated to one of the remaining groups according to the *MSE*. Default is 0.05.
const_coef	a character vector containing the variable names of explanatory variables that enter with time-constant coefficients.
kappa	the a non-negative weight used to obtain the adaptive penalty weights. Default is 2.
max_iter	the maximum number of iterations for the *ADMM* estimation algorithm. Default is $5 * 10^4$.

`tol_convergence` the tolerance limit for the stopping criterion of the iterative *ADMM* estimation algorithm. Default is $1 * 10^{-10}$.

`tol_group` the tolerance limit for within-group differences. Two individuals are assigned to the same group if the Frobenius norm of their coefficient vector difference is below this threshold. Default is $1 * 10^{-3}$.

`rho` the tuning parameter balancing the fitness and penalty terms in the IC that determines the penalty parameter λ. If left unspecified, the heuristic $\rho = 0.07\frac{\log(NT)}{\sqrt{NT}}$ of Mehrabani (2023, sec. 6) is used. We recommend the default.

`varrho` the non-negative Lagrangian *ADMM* penalty parameter. For the employed penalized sieve estimation *PSE*, the ϱ value is trivial. We recommend the default 1.

`verbose` logical. If `TRUE`, helpful warning messages are shown. Default is `TRUE`.

`parallel` logical. If `TRUE`, certain operations are parallelized across multiple cores. Default is `TRUE`.

`...` ellipsis

`object` of class `tvpagfl`.

`x` of class `tvpagfl`.

Details

Consider the grouped time-varying panel data model

$$y_{it} = \gamma_i + \beta_i'(t/T)x_{it} + \epsilon_{it}, \quad i = 1, \dots, N,\ t = 1, \dots, T,$$

where y_{it} is the scalar dependent variable, γ_i is an individual fixed effect, x_{it} is a $p \times 1$ vector of explanatory variables, and ϵ_{it} is a zero mean error. The coefficient vector $\beta_i(t/T)$ is subject to the latent group pattern

$$\beta_i\left(\frac{t}{T}\right) = \sum_{k=1}^{K} \alpha_k\left(\frac{t}{T}\right)\mathbf{1}\{i \in G_k\},$$

with $\cup_{k=1}^{K} G_k = \{1, \dots, N\}$, $G_k \cap G_j = \emptyset$ and $\|\alpha_k - \alpha_j\| \neq 0$ for any $k \neq j$, $k = 1, \dots, K$.

The time-varying coefficient functions are estimated as polynomial B-splines using the penalized sieve-technique. To this end, let $B(v)$ denote a $M + d + 1$ vector basis functions, where d denotes the polynomial degree and M the number of interior knots. Then, $\beta_i(t/T)$ and $\alpha_k(t/T)$ are approximated by forming linear combinations of the basis functions $\beta_i(t/T) \approx \pi_i' B(t/T)$ and $\alpha_i(t/T) \approx \xi_k' B(t/T)$, where π_i and ξ_i are $(M + d + 1) \times p$ coefficient matrices.

The explanatory variables are projected onto the spline basis system, which results in the $(M + d + 1)p \times 1$ vector $z_{it} = x_{it} \otimes B(v)$. Subsequently, the DGP can be reformulated as

$$y_{it} = \gamma_i + z_{it}'\text{vec}(\pi_i) + u_{it},$$

where $u_{it} = \epsilon_{it} + \eta_{it}$ and η_{it} reflects a sieve approximation error. We refer to Su et al. (2019, sec. 2) for more details on the sieve technique.

Inspired by Su et al. (2019) and Mehrabani (2023), the time-varying PAGFL jointly estimates the functional coefficients and the group structure by minimizing the criterion

$$Q_{NT}(\boldsymbol{\pi}, \lambda) = \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} (\tilde{y}_{it} - \tilde{z}_{it}' \text{vec}(\pi_i))^2 + \frac{\lambda}{N} \sum_{i=1}^{N-1} \sum_{j>i}^{N} \dot{\omega}_{ij} \|\pi_i - \pi_j\|$$

with respect to $\boldsymbol{\pi} = (\text{vec}(\pi_i)', \ldots, \text{vec}(\pi_N)')'$. $\tilde{a}_{it} = a_{it} - T^{-1} \sum_{t=1}^{T} a_{it}$, $a = \{y, z\}$ to concentrate out the individual fixed effects γ_i. λ is the penalty tuning parameter and $\dot{w}_{ij}$ denotes adaptive penalty weights which are obtained by a preliminary non-penalized estimation. $\|\cdot\|$ represents the Frobenius norm. The solution criterion function is minimized via the iterative alternating direction method of multipliers (*ADMM*) algorithm proposed by Mehrabani (2023, sec. 5.1).

Two individuals are assigned to the same group if $\|\text{vec}(\hat{\pi}_i - \hat{\pi}_j)\| \leq \epsilon_{\text{tol}}$, where ϵ_{tol} is determined by `tol_group`. Subsequently, the number of groups follows as the number of distinct elements in $\hat{\boldsymbol{\pi}}$. Given an estimated group structure, it is straightforward to obtain post-Lasso estimates $\hat{\boldsymbol{\xi}}$ using group-wise least squares (see `grouped_tv_plm`).

We recommend identifying a suitable λ parameter by passing a logarithmically spaced grid of candidate values with a lower limit close to 0 and an upper limit that leads to a fully homogeneous panel. A BIC-type information criterion then selects the best fitting λ value.

In case of an unbalanced panel data set, the earliest and latest available observations per group define the start and end-points of the interval on which the group-specific time-varying coefficients are defined.

Value

An object of class `tvpagfl` holding

`model`	a `data.frame` containing the dependent and explanatory variables as well as cross-sectional and time indices,
`coefficients`	let $p^{(1)}$ denote the number of time-varying coefficients and $p^{(2)}$ the number of time constant parameters. A `list` holding (i) a $T \times p^{(1)} \times \hat{K}$ array of the post-Lasso group-specific functional coefficients and (ii) a $K \times p^{(2)}$ matrix of time-constant post-Lasso estimates.
`groups`	a `list` containing (i) the total number of groups $\hat{K}$ and (ii) a vector of estimated group memberships $(\hat{g}_1, \ldots, \hat{g}_N)$, where $\hat{g}_i = k$ if i is assigned to group k,
`residuals`	a vector of residuals of the demeaned model,
`fitted`	a vector of fitted values of the demeaned model,
`args`	a `list` of additional arguments,
`IC`	a `list` containing (i) the value of the IC, (ii) the employed tuning parameter λ, and (iii) the *MSE*,
`convergence`	a `list` containing (i) a logical variable if convergence was achieved and (ii) the number of executed *ADMM* algorithm iterations,
`call`	the function call.

An object of class `tvpagfl` has `print`, `summary`, `fitted`, `residuals`, `formula`, `df.residual` and `coef` S3 methods.

Author(s)

Paul Haimerl

References

Mehrabani, A. (2023). Estimation and identification of latent group structures in panel data. *Journal of Econometrics*, 235(2), 1464-1482. doi:10.1016/j.jeconom.2022.12.002.

Su, L., Wang, X., & Jin, S. (2019). Sieve estimation of time-varying panel data models with latent structures. *Journal of Business & Economic Statistics*, 37(2), 334-349. doi:10.1080/07350015.2017.1340299.

Examples

```
# Simulate a time-varying panel with a trend and a group pattern
set.seed(1)
sim <- sim_tv_DGP(N = 10, n_periods = 50, intercept = TRUE, p = 1)
df <- data.frame(y = c(sim$y))

# Run the time-varying PAGFL
estim <- tv_pagfl(y ~ ., data = df, n_periods = 50, lambda = 10, parallel = FALSE)
summary(estim)
```

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